Electron-microwave interactions

1972 J. Phys. A: Gen. Phys. 5 516

(http://iopscience.iop.org/0022-3689/5/4/008)

J. Phys. A: Gen. Phys., Vol. 5, April 1972. Printed in Great Britain

Electron–microwave interactions

F WATT†

Research Laboratory for Solid State Physics, School of Physics, University of Newcastle upon Tyne, Newcastle upon Tyne, UK

MS received 19 August 1971

Abstract. The equations of motion of a relativistic electron interacting with a microwave electric field while undergoing circular motion in a homogeneous magnetic field have been solved numerically with the aid of a fast computer. The results serve to explain the phenomena of wave amplification and maser action using classical electron trajectories and confirm the phase bunched acceleration of low energy electrons.

1. Introduction

Theoretical studies involving the interaction between microwaves and free electrons have proved difficult except in special cases. Only under limited conditions have the equations of motion been solved explicitly.

Consoli and Mourier (1963) calculated the maximum energy attained by an electron undergoing cyclotron acceleration by microwaves for the case where the electron cyclotron frequency is initially equal to the frequency of the microwave field. Hakkenberg and Weenink (1964) have shown that this energy maximum is increased when the electron is injected into the interaction region such that the electron cyclotron frequency is initially greater than the microwave frequency. In both cases the additional condition was made that the electrons were accelerated from zero energy.

When the electrons are initially relativistic, the problems associated with the interaction become more difficult. Bohm and Foldy (1946) recognized that in the relativistic case the energy gain mechanism is dependent on the initial phase of the electron with respect to the microwave field. They provided an interpretation of the phase equation of the electron by means of a simple mechanical model, the biased pendulum. E E Schneider (1971, private communication) has shown that by using this model the energy equation can be solved explicitly in terms of elliptical integrals for the case of synchronous injection. Roberts and Buchsbaum (1964) used a pseudopotential approach to show that the energy equation for the relativistic electron initially out of synchronism can also be expressed explicitly. Because of the impractical aspects of solving such a formidable equation however, they considered the case for synchronous injection only.

This paper shows some of the results obtained from solving numerically the equations of motion, not only for synchronous injection but also for the more complicated off-resonance case.

†Now at Physical Chemistry Laboratory, Oxford University, South Parks Rd, Oxford, UK.

2. Mathematical formulation of the motion

The relativistic equation of motion for an electron in a homogeneous magnetic field under the influence of a rotating microwave electric field is

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -e\left(\boldsymbol{E}+\frac{\boldsymbol{p}}{\gamma m_0}\times \boldsymbol{B}\right)$$

where $\boldsymbol{p}$ is the relativistic momentum

$$\gamma = \left(1+\frac{p^2}{m_0 c^2}\right)^{1/2}$$

and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the microwave electric field and the homogeneous magnetic field respectively, given as

$$\boldsymbol{E} = (E_0 \cos \omega t, E_0 \sin \omega t, 0)$$

$$\boldsymbol{B} = (0, 0, B).$$

In this configuration the homogeneous magnetic field is perturbed by a microwave magnetic field. It is assumed that this perturbation is negligible.

Following Hakkenburg and Weenink (1964), it is convenient to use cylindrical coordinates (p, ϕ) in momentum space. The equations of motion in a plane perpendicular to the magnetic field direction are then

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -eE\cos(\omega t-\phi)$$

$$p\frac{\mathrm{d}\phi}{\mathrm{d}t} = -eE\sin(\omega t-\phi)+\frac{eB}{m_0\gamma}p.$$

Rewriting the above equations, we have

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -eE\cos\chi \tag{1}$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}t} = \frac{eB}{m_0\gamma}-\omega+\frac{eE}{p}\sin\chi \tag{2}$$

where $\chi = \phi-\omega t$. The kinetic energy T of the electron is given by

$$T = (p^2c^2+T_0^2)^{1/2}-T_0$$

where T_0 is the rest energy.

3. Computer program

A combination of subroutines was developed to program an IBM 360 computer to calculate numerically the information required on the electron motion. Subroutine DRKGS (IBM systems reference library) was used to solve the two differential equations (1) and (2) and print out values of p and χ for selected operating parameters. This subroutine is a fourth order integration procedure and uses the Runge–Kutta method for obtaining the solutions of initial value problems. A histogram plotting routine was included in the program to aid the handling of the results.

4. Preliminary tests

The equations of motion of a low energy (0·1 eV) electron injected in synchronism with an X band (9·3 GHz) microwave electric field of strength 600 V m^{-1} were solved numerically for a selection of initial phase angles χ_0. The variation of phase χ with time is shown in figure 1. It is seen that a rapid change in phase occurs during the first

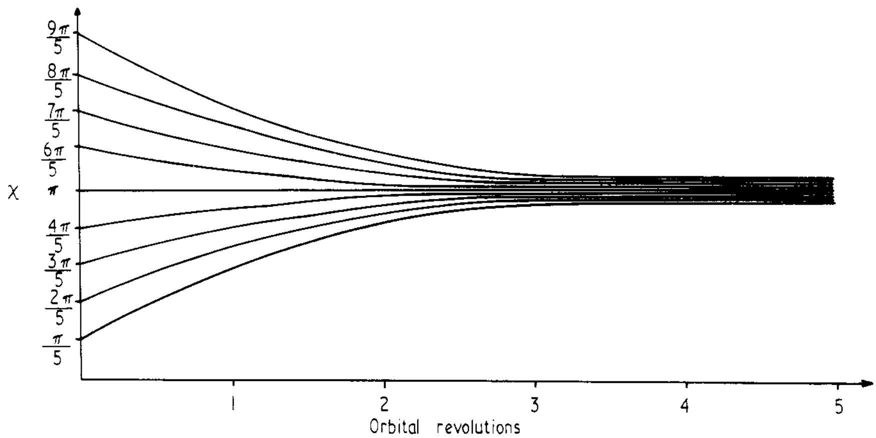

Figure 1. A plot of the phase of a low energy electron with respect to the rotating electric field against number of orbital revolutions.

few orbital revolutions. The electron, within a short time from injection, is focused about the phase $\chi = \pi$, corresponding to the maximum accelerating field. This result is well known. At low energies the magnetic forces are small and therefore the motion is dominated by the rotating electric field. The previously circular trajectory is violently disrupted and a considerable change in phase may occur.

The low energy electron injected in synchronism with the rotating electric field therefore gains energy at a rate which is independent of its initial phase. The gain in energy however is eventually limited by the effects of the increased mass as predicted by the special theory of relativity. The associated decrease in the electron cyclotron frequency ω_c, where $\omega_c = Be/m$, causes a phase lag which eventually results in the deceleration of the particle. The maximum energy which the particle can attain is dependent on the rotating electric field strength E. Figure 2 shows the energy profiles of the electrons for several different field strengths. The maximum energy to which the electrons are accelerated T_m and the time taken in each case τ_m confirm equations derived by Consoli and Mourier (1963) which predict a $\frac{2}{3}$ power dependence between T_m and E, and τ_m and E.

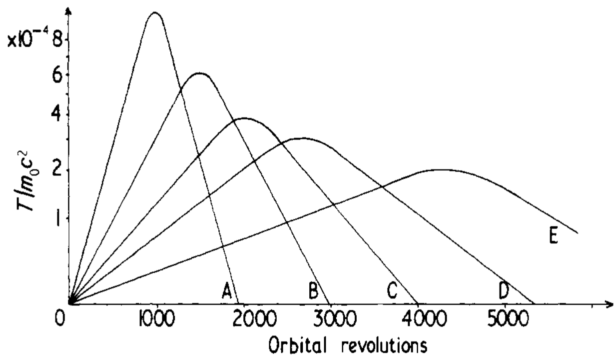

Figure 2. A plot of the kinetic energy of a low energy electron initially in synchronism with the microwave electric field against the number of orbital revolutions for several field strengths: A 10^3 V m^{-1}; B 5×10^2 V m^{-1}; C 3×10^2 V m^{-1}; D 2×10^2 V m^{-1}; E 10^2 V m^{-1}.

5. Results

The following results refer to an electron with an energy of 5 keV interacting with an X band microwave field of strength 100 V m^{-1}.

5.1. The motion of a medium energy electron injected in synchronism with the rotating electric field

This is the case where the electron cyclotron frequency is initially equal to the frequency of the rotating microwave field. Figure 3 shows the energy and phase profiles of the

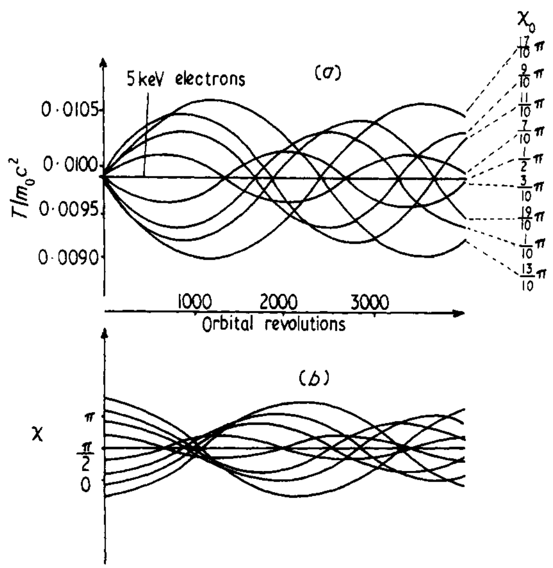

Figure 3. The motion of an electron injected in synchronism with the microwave electric field for a selection of initial phases. (*a*) Plot of energy against number of orbital revolutions. (*b*) Plot of phase χ against number of orbital revolutions.

electron for a selection of initial phase angles. It is seen that, unlike the low energy case, the motion is dependent on the phase of the electron at injection.

When the electron is injected in synchronism with an accelerating field, that is $-\pi/2 < \chi_0 < \pi/2$, the electron initially experiences a phase slip as a result of the gain in kinetic energy. The energy gain continues until the electron phase is sufficiently retarded for the electron to experience a decelerating field. Similarly when the electron is injected in synchronism within a decelerating field, that is $\pi/2 < \chi_0 < 3\pi/2$, the particle experiences a loss in energy and an associated advance in phase. This loss of energy continues until the electron phase is sufficiently advanced for the electron to experience an accelerating field.

It is seen in figure 3 that the electron energy oscillates about the injection value and the phase oscillates about $\chi = \pi/2$. An exception to this is when $\chi_0 = \pi/2$, corresponding to injection at right angles to the rotating electric field. The electron experiences neither acceleration nor deceleration and the synchronism condition is maintained throughout the motion.

From these results, the energy exchange at resonance between the microwave field and relativistic electrons can be predicted. Although the rate of energy transfer depends on the phase position occupied by the electrons, it can be seen that the energy gained by electrons injected with phase χ_0 is equal to the energy lost to the field by electrons starting at phase $\pi - \chi_0$. For an initial random phase distribution therefore, the energy transfer and hence the power absorbed by the electrons from the microwave field is zero at resonance.

5.2. The motion of a medium energy electron injected out of synchronism with the rotating electric field

The case considered first is where the energy of the electron is lower than that required for resonance and hence the electron cyclotron frequency is initially greater than the frequency of the microwave field. Figure 4 shows the energy and phase profiles of such an electron for a selection of input phases.

Suppose the electron is injected into an acceleration phase zone, $-\pi/2 < \chi_0 < \pi/2$. As discussed previously the electron accelerates until the phase slippage associated with the relativistic mass increase causes it to move into a deceleration phase zone. In this case however the rate of phase slippage is retarded by the difference between the cyclotron and microwave frequencies. Initially therefore the electron spends more time in the acceleration zone than it would otherwise under synchronous injection conditions and hence experiences a greater increase in kinetic energy.

For an electron injected into the deceleration zone, $-\pi/2 < \chi_0 < 3\pi/2$, the phase advance associated with the energy loss is enhanced by the difference between the cyclotron and microwave frequencies. Initially therefore the electron spends less time in the deceleration zone than it would at resonance and hence a reduction in the amount of energy lost is experienced.

It is seen in figure 4 that if the electron is injected into the interaction region with an initial phase angle between $\frac{13}{10}\pi$ and $\frac{17}{10}\pi$ then the synchronism condition is never fulfilled. In these cases the phase advance experienced by the electron arising from the difference between the cyclotron and microwave frequencies dominates any phase slippage suffered when the electron passes through the acceleration phase zone. The effect of this is that the electron energy never reaches the value necessary for synchronous motion and the electron cyclotron frequency always remains greater than the frequency

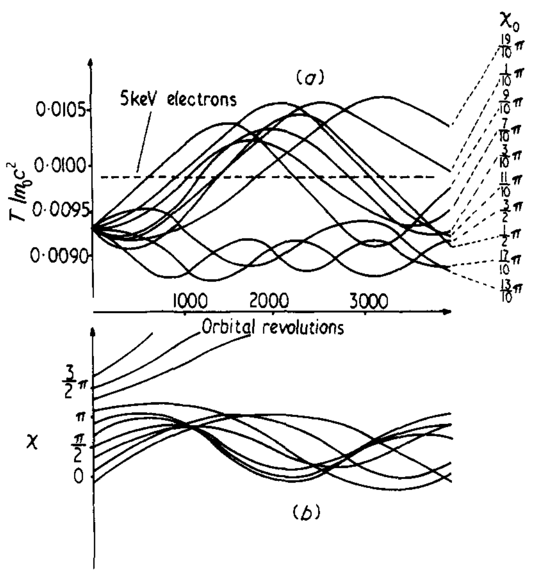

Figure 4. The motion of an electron injected out of synchronism with the microwave electric field for a selection of initial phases. (*a*) Plot of energy against number of orbital revolutions. (*b*) Plot of phase χ against number of orbital revolutions.

of the microwave field. Obviously when the difference between the two frequencies at injection is greater, then the fraction of input phases for which this nonsynchronous type of motion prevails is correspondingly larger.

For an initial random phase distribution, the electrons which are phase locked in time to the microwave field experience a net energy gain. Hence in the case considered above, $\omega_c > \omega$, there is a net absorption of power by the electrons from the microwave field.

The second case considered is where the energy of the electron is higher than that required for resonance and the electron cyclotron frequency is initially less than the frequency of the microwave field. Figure 5 shows the energy and phase profiles of such an electron for a selection of input phases.

As expected, the difference between the cyclotron and microwave frequencies initially causes the electron to remain longer in the deceleration phase zone and hence lose more energy than it would otherwise at resonance. The electron which is injected into an acceleration phase region initially spends less time in this region than it would under synchronous injection conditions. Again it can be seen that for certain initial phase angles the synchronism condition is never fulfilled due to the masking of the phase advance suffered by an electron passing through a deceleration phase zone by the difference between the cyclotron and microwave frequencies.

For an initial random phase distribution, the electrons which are phase locked in time to the microwave field experience a net energy loss. Hence for the case $\omega_c < \omega$ the microwave field absorbs energy from the electrons.

Figure 5 also shows that these electrons are bunched together in phase after approximately 1000 orbital revolutions. This phase bunching occurs while the majority of the

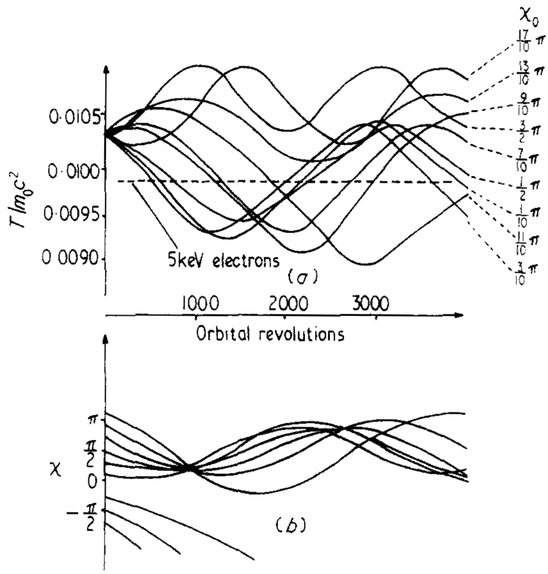

Figure 5. The motion of an electron injected out of synchronism with the microwave electric field for a selection of initial phases. (*a*) Plot of energy against number of orbital revolutions. (*b*) Plot of phase χ against number of orbital revolutions.

electrons are in close synchronism with the decelerating field, thereby leading to coherent emission at the expense of the electron energy.

6. Summary and concluding remarks

The results of this work provide insight into several features of cyclotron resonance phenomena and associated electron–microwave interactions.

The computer plots of the electron motion show the phase bunched cyclotron acceleration of low energy electrons and confirm equations predicting the maximum energy attainable by these electrons. For a relativistic electron the motion is seen to be dependent on the initial phase of the electron with respect to the microwave field. The wave amplification and stimulated emission observed in practice is shown to be caused by the phase bunching of electrons. The shape of the resonance absorption curve of medium energy electrons is predicted qualitatively from the electron motion and agrees with calculations derived by both quantum mechanical and classical methods (Schneider 1959, 1960).

This type of numerical approach to the problems of electron–microwave interactions may be useful in explaining electron maser saturation effects and for investigating new methods of monochromatizing high energy electron beams of wide energy distribution.

Acknowledgments

The author wishes to thank Dr P K W Vinsome for valuable assistance in computing techniques and acknowledge the financial support of the SRC of Great Britain.

References

Bohm D and Foldy L 1946 *Phys. Rev.* **70** 249–58
Consoli T and Mourier G 1963 *Phys. Lett.* **7** 247–9
Hakkenberg A and Weenink M P H 1964 *Physica* **30** 2147–52
Roberts C S and Buchsbaum S J 1964 *Phys. Rev.* **135** A381–9
Schneider J 1959 *Phys. Rev. Lett.* **2** 504–5
—— 1960 *Z. Naturf.* **15** 484–9